Week 11

Generalized Assignment Problem

Generalized Assignment Problem

Formulation

m agents, indexed with i
n tasks, indexed with j

$$\text{maximize} \quad \sum_{i=1}^{m} \sum_{j=1}^{n} p_{ij} x_{ij}.$$

find an assignment in which agents do not exceed their budget
and total profit of the assignment is maximized

$$\text{subject to} \quad \sum_{j=1}^{n} w_{ij} x_{ij} \leq t_i \qquad i = 1, \ldots, m;$$

each agent i has a budget/capacity

$$\sum_{i=1}^{m} x_{ij} = 1 \qquad j = 1, \ldots, n;$$

each task j has to be assigned

$$x_{ij} \in \{0, 1\} \qquad i = 1, \ldots, m, \quad j = 1, \ldots, n$$

Data
Profits

```sql
SELECT * FROM `profits`
```

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AgentID	TaskID	profit
1	1	27
2	1	37
3	1	13
4	1	38
5	1	12
6	1	26
7	1	33
8	1	35
9	1	27
10	1	28
11	1	11
12	1	37
13	1	37
14	1	27
15	1	29
16	1	12
17	1	29
18	1	34
19	1	36
20	1	38
1	2	25
2	2	40
3	2	27
4	2	13
5	2	27

Data
Capacity Consumption

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AgentID	TaskID	CapacityRequired
1	1	71
2	1	61
3	1	83
4	1	83
5	1	86
6	1	52
7	1	61
8	1	52
9	1	66
10	1	51
11	1	39
12	1	77
13	1	68
14	1	80
15	1	48
16	1	32
17	1	75
18	1	32
19	1	52
20	1	30
1	2	83
2	2	62
3	2	94
4	2	80
5	2	97

Data
Available Capacity

```
SELECT * FROM `capacities`
```

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AgentID	Capacity
1	138
2	101
3	149
4	123
5	129
6	139
7	154
8	114
9	135
10	158
11	133
12	142
13	156
14	137
15	114
16	118
17	127
18	161
19	155
20	136

Data
Restrictions

```sql
SELECT * FROM `properties`
```

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AgentID	Property
1	full
2	partial
3	full
4	partial
5	full
6	full
7	full
8	full
9	full
10	partial
11	full
12	full
13	full
14	full
15	partial
16	full
17	full
18	partial
19	full
20	partial

Modified Generalized Assignment Problem

Formulation

m agents, indexed with i
n tasks, indexed with j

$$\text{maximize} \quad \sum_{i=1}^{m} \sum_{j=1}^{n} p_{ij} x_{ij}.$$

find an assignment in which agents do not exceed their budget
and total profit of the assignment is maximized

$$\text{subject to} \quad \sum_{j=1}^{n} w_{ij} x_{ij} \leq t_i \qquad i = 1, \ldots, m;$$

each agent i has a budget/capacity

$$\sum_{i=1}^{m} x_{ij} = 1 \qquad j = 1, \ldots, n;$$

each task j has to be assigned

$$x_{ij} \in \{0, 1\} \qquad i \in F, j = 1, \cdots, n$$
$$0 \leq x_{ij} \leq 1 \qquad i \in P, j = 1, \cdots, n$$